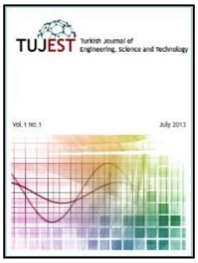
Turkish Journal of Engineering, Science and Technology

journal homepage: www.tujest.com



Velocity field analysis of a ceiling fan

Mohammed A. Azim[a,*]

[a]*Assistant Professor, Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Bangladesh*

Article Info

Article history:
Received November 23, 2013
Accepted December 05, 2013
Available online February 13, 2014

Keywords:
Ceiling fan,
Blade angle,
Pressure gradient,
Vector analysis,
Axial velocity

Abstract

A mathematical analysis in vector notation for the velocity field of a ceiling (axial flow) fan has lead to an expression for its axial velocity distribution. In the analysis, momentum exerted by the fan on the fluid is approximated to be proportional to the momentum that a rotating disk exerts. Furthermore, the momentum exerted by the fan is assumed to be reduced by the amount of momentum absorbed due to the fluid mass entangled with the blade for its curved width. Axial velocity distribution obtained from the present study is found to be in good agreement with the existing experimental and numerical simulation data that validates the assumptions made herein for the velocity field analysis of the fan.

1. Introduction

Ceiling fans have widespread use in hot or tropical climate zones for providing cooling and comfort. It consumes relatively low amount of energy in comparison to air-conditioning units. Complementarily using ceiling fans as additional air circulating devices in air-conditioned rooms can often lead to the same comfort level at an elevated temperature setting, which means a potential saving of energy [1]. Recent study shows that the distribution of a constant airflow at the outlet of an air moving device is quite different from that of a simulated natural wind, and the fluctuation of airflow is one of the main factors that affects indoor comfort [2]. According to some researchers, the fluctuating airflow brings more cooling effect to the human bodies, especially the airflow with a frequency similar to that of natural wind [3-5]. Hence, investigation on the performance and flow characteristics of ceiling fans is of interests, even in these days air-conditioning is preferable to operating a ceiling fan in modern office or residential buildings.

The less conventional method of axial flow fan analysis considers a whirl velocity distribution downstream from the rotor without a free vortex and beyond some distance downstream the axial velocity varies radially [6]. The flow field near the blades is fairly two-dimensional axisymmetric and after a short downstream distance is dominated by the axial flow. This axial flow decreases in the downstream due to fluid friction and spreads radially outward. Figure 1 shows the isovels of axial velocity of a ceiling fan after Aynsley [7] that illustrates the spread of the axial flow very nicely. There the flow remains almost within the rotational radius of the fan for a large downstream distance and spreads largely while close to the floor.

The streamlines due to V_θ -velocity that are close to the blade across its width (*abc*) is shown in Fig.2. The upper streamlines across the blade width are pushed away from the blade causing an increase in pressure and the lower streamlines are pushed closer causing a decrease in pressure, thus produces a negative downstream pressure gradient resulting in a through axial flow without any significant radial flow. A rotating disk in fluid acts like a centrifugal fan by throwing fluid radially outward and drawing other fluid axially toward it to be thrown out in turn [8]. It is obvious that the flow due to rotating disk is different from that of ceiling fan but their mechanism of momentum transfer by viscosity to the fluid may be similar at the limiting condition of insignificant radial flow in an axial flow fan.

*Corresponding Author:
A. Azimi, E-mail: azim@me.buet.ac.bd

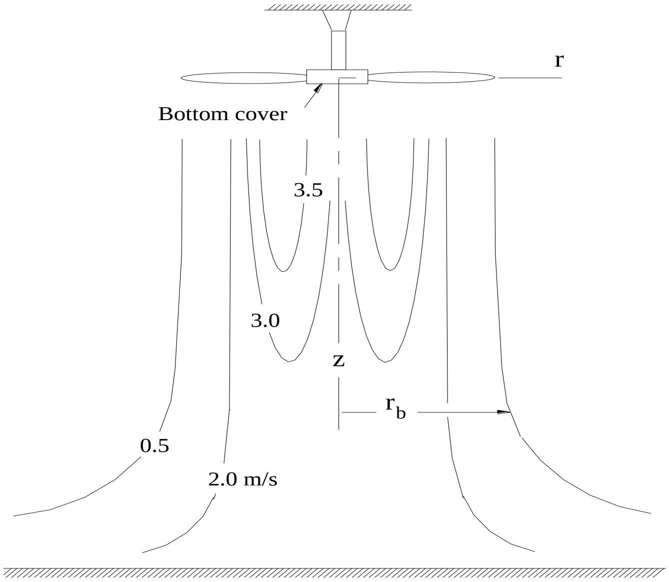


Fig. 1 Schematic of a ceiling fan and its isovels for axial velocity [7].

In this paper, Section 2 presents a mathematical analysis of the flow field of a ceiling fan leading to an expression for its axial velocity distribution. Section 3 presents a comparison of the obtained velocity distribution from the present calculation with the available experimental and computational fluid dynamics (CFD) simulation data.

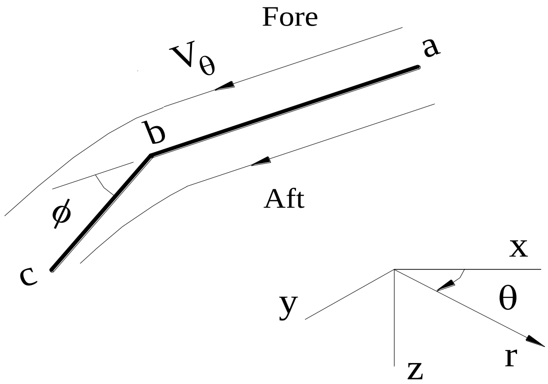


Fig. 2 Blade cross-section of a ceiling fan and geometrical axis.

2. Mathematical Analysis

The mathematical analysis for the flow field of the fan is organized as vector analysis, circumferential momentum and axial velocity distribution in this section.

2.1. Vector analysis

In a differentiable divergence free field, curl of $\bar{r} \times \bar{V}$ reduces to

$$\nabla \times \left(\bar{r} \times \bar{V}\right) = -3\bar{V} \tag{1}$$

where $\bar{r}$ and $\bar{V}$ are the position and velocity vectors (Appendix). Equation (1) implies that the curl of momentum vector is thrice of the velocity vector, and one may write

$$\nabla \times \overline{M} = -3\rho \overline{V} \tag{2}$$

where $\overline{M} = \rho \bar{r} \times \overline{V}$ is the angular momentum vector per unit volume and ρ is the fluid density. Integration of Eq. (2) over any closed surface provides

$$\int \nabla \times \overline{M} \cdot d\bar{s} = -\int 3\rho \overline{V} \cdot d\bar{s} \tag{3}$$

where $\bar{s}$ is the area vector. By the application of Stokes's theorem, Eq. (3) becomes

$$\oint \overline{M} \cdot d\bar{\ell} = -3\rho \int \overline{V} \cdot d\bar{s} \tag{4}$$

where $\bar{\ell}$ is the line vector. The vectors $\overline{M}$, $d\bar{\ell}$ and $\overline{V}$ in cylindrical co-ordinates (*r*, θ, *z*) are as follows:

$$\overline{M} = \bar{i}_r M_r + \bar{i}_\theta M_\theta + \bar{i}_z M_z \tag{5}$$

$$d\bar{\ell} = \bar{i}_r dr + \bar{i}_\theta r d\theta + \bar{i}_z dz \tag{6}$$

$$\overline{V} = \bar{i}_r V_r + \bar{i}_\theta V_\theta + \bar{i}_z V_z \tag{7}$$

where $\bar{i}_r, \bar{i}_\theta, \bar{i}_z$ are the unit vectors. Using Eqs. (5) and (6) in Eq. (4), one obtains

$$\oint M_r dr + \oint M_\theta r d\theta + \oint M_z dz = -3\rho \int \overline{V} \cdot d\bar{s}\,. \tag{8}$$

The closed integrals $\oint M_r dr$ and $\oint M_z dz$ do not exist, being the contours in *r* and *z*-directions are not closed within the cylindrical flow volume, and thus Eq. (8) reduces to

$$\oint M_\theta r d\theta = -3\rho \int \overline{V} \cdot d\bar{s}\,. \tag{9}$$

As axial velocity V_z is dominant beyond some distance downstream [6] and circumferential momentum M_θ is independent of θ due to the radial symmetry, Eq. (9) becomes

$$M_\theta r = -3\rho \int V_z r dr\,. \tag{10}$$

2.2. Circumferential Momentum

The thickness δ of the fluid layer carried by the blades due to viscosity adheres to the fluid layer entrapped due to the blade angle. There the radial component of the shear stress is very small compared to the circumferential stress owing to negligible radial flow near the blade. However, the thickness δ remains proportional to $\sqrt{\nu/\omega}$ where ν is the fluid viscosity and ω is the angular speed of the fan. M_θ exerted by the fan on the fluid is the product of the shear stress at blade surface τ_w, the blade area on which the stress is acting and the radial distance at which the stress is acting that is

$$M_\theta \approx \tau_w r^3. \tag{11}$$

Using $\tau_w \approx \rho v r\omega / \delta$ from the analysis of flow near a rotating disk [9] and $\delta \approx \sqrt{v/\omega}$ as mentioned above, Eq. (11) becomes

$$M_\theta \approx \rho\omega\sqrt{v\omega}\, r^4. \tag{12}$$

A part of the momentum in Eq. (12) is used up to rotate the fluid mass $\rho\pi h sin\phi\left(r^2 - r_1^2\right)$ that adheres to the aft of the ceiling fan due to the blade angle where r_1 is radius of the bottom cover of the fan (Fig. 1), h is the fraction (bc) of the blade width and ϕ is the blade angle (Fig. 2). An axial flow through the (r, θ, 0) plane without any significant radial flow implies a uniform streamwise pressure gradient about the same plane without any significant radial variation in pressure. This streamwise pressure gradient over (r, θ, 0) plane can be made uniform either by varying h or ϕ along the blade length. Considering constant value of ϕ and varying $h=a_1(1-a_2r)$, the momentum absorbed by the rotating fluid mass in the form $I\omega^2$ becomes

$$\approx \rho a_1\left(1-a_2r\right)sin\phi\left(r^2 - r_1^2\right)r^2\omega^2 \tag{13}$$

where I is the mass moment of inertia. Hence, the net momentum exerted on the fluid by the fan becomes

$$M_\theta = C_1\rho\omega\sqrt{v\omega}\, r^4 - C_2\rho a_1\left(1-a_2r\right)sin\phi\left(r^2 - r_1^2\right)r^2\omega^2 \tag{14}$$

where C_1 and C_2 are the constants of proportionality. An axial flow through the (r, θ, 0) plane induced by the downstream pressure gradient causes a volume flow at the rate of $\approx \omega r_o^3$ where r_o is the blade length from the rotational axis. Using this volume flow rate, the circumferential momentum per unit volume from Eq. (14) may be expressed as

$$M_\theta = C_1'\rho\omega\sqrt{v\omega}\, r^4 / \omega r_o^3 - C_2'\rho a_1\left(1-a_2r\right)sin\phi\left(r^2 - r_1^2\right)r^2\omega^2 / \omega r_o^3. \tag{15}$$

2.3. Axial Velocity Distribution

On differentiation once, Eq. (10) takes the form

$$\frac{\partial}{\partial r}\left(M_\theta r\right) = -3\rho r V_z. \tag{16}$$

At rotational axis, M_θ becomes zero while V_z is not zero that necessitates to write V_z with reference to the centerline axial velocity V_c, and thus substitution of Eq. (15) into Eq. (16) results in

$$V_z - V_c = -5C_1'\sqrt{v\omega}\, r^3 / 3r_o^3 + C_2'a_1\omega sin\phi\left(-6a_2r^4 + 5r^3 + 4a_2r_1^2r^2 - 3r_1^2r\right)/ 3r_o^3. \tag{17}$$

Normalizing the variables in Eq. (17) by $R = r / r_b$ and $V_n = \left(V_z - V_c\right)/\left(V_m - V_c\right)$ where r_b is the radius of spread of the flow and V_m is the maximum axial velocity on a radial plane, one obtains

$$V_n = -5C_1'\sqrt{v\omega}\, r_b^3R^3 / 3r_o^3\left(V_m - V_c\right) + C_2'a_1r_b\,\omega sin\phi\left(-6a_2r_b^3R^4 + 5r_b^2R^3 + 4a_2r_1^2r_b\,R^2 - 3r_1^2R\right)/ 3r_o^3\left(V_m - V_c\right). \tag{18}$$

For a ceiling fan with known specifications, Eq. (18) may be written as

$$V_n = -C_1''r_b^3R^3/(V_m - V_c) + C_2''r_b\left(-6a_2r_b^3R^4 + 5r_b^2R^3 + 4a_2r_1^2r_b R^2 - 3r_1^2R\right)/(V_m - V_c). \quad (19)$$

Considering $r_b \approx r_o$ in the downstream before reaching the floor effect, (V_m - V_c) as constant on a radial plane and $a_2 = 1/r_o$, Eq. (19) with the typical specifications $r_o = 65\ cm$ and $r_1 = 10\ cm$ reduces to

$$V_n = -AR^3 + B\left(-2.54R^4 + 2.11R^3 + 0.04R^2 - 0.03R\right) \quad (20)$$

where *A* and *B* are constants. Experiments, e.g. Parker et al. [10] and Momoi et al. [11], show that maximum axial velocity over a radial plane occurs at $R \cong 2/3$ yielding the conditions

$$V_{n\,max} = 1, \quad \partial V_n/\partial R = 0. \quad (21)$$

Using the conditions in Eq. (21), the constants in Eq. (20) are evaluated as $A = -0.7$ and $B = 6.55$. Hence, the axial velocity distribution of a ceiling fan in the downstream with the given specification is

$$V_n = -16.6R^4 + 14.5R^3 + 0.26R^2 - 0.2R. \quad (22)$$

3. Results and Discussion

In order to analyze the velocity field of an axial flow fan, the expression for M_θ is written in the light of the results of a rotating disk with an additional consideration for the momentum due to the fluid mass entrapped by the curved blade width. Figure 1 shows a schematic of a ceiling fan, isovels of its axial velocity after Aynsley [7] and its nomenclature. The isovels show that maximum axial velocity over the entire flow domain occurs close to the (*r, θ, 0*) plane. Figure 2 illustrates the details of the fan's blade, orientation of the streamlines due to V_θ -velocity and their nomenclature.

Figure 3 shows the radial variation of normalized axial velocity which is calculated by Eq. (22). In the figure, axial velocity distribution from the present study is compared with Momoi et al. [11] velocity data both from experiment and CFD

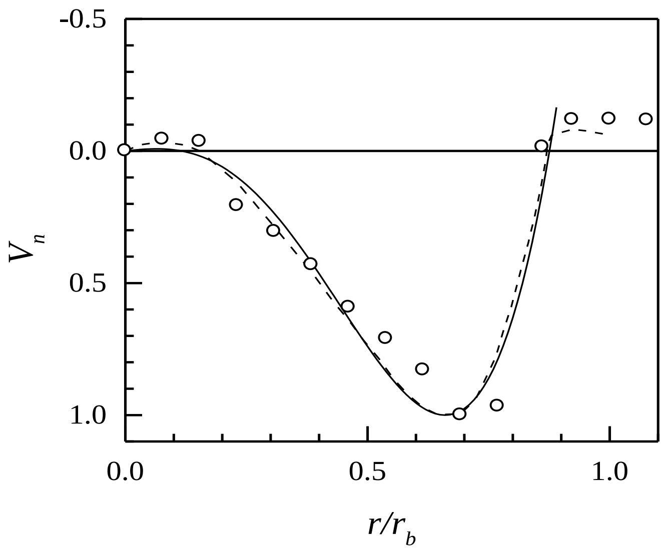


Fig. 3 Normalized axial velocity distribution.
——— Present, - - - - - CFD [11], ○ Expt [11]

simulation at *z = 20 cm*. The experiment was carried out in a large room for ceiling fan with radius of *65 cm* at a speed of *160 rpm*. Measurements were performed for *30 s* at *10 Hz* sampling frequencies using a three-dimensional ultrasonic anemometer and the average of each air velocity component was calculated. CFD simulation of the experimented space was performed using the standard $k-\varepsilon$ turbulence model and SIMPLEC algorithm. Air velocity data measured near the ceiling fan were used as the boundary conditions in the simulation. Comparison in Fig. 3 shows that axial velocity distribution obtained from the present mathematical analysis is in good agreement with that of the experimental as well as CFD data for a ceiling fan.

4. Conclusions

The mathematics of the flow field of a ceiling fan resolves into simple form by the use of vector analysis for predominant axial flow. This paper assumes that momentum exerted by the fan can be approximated as proportional to the momentum exerted by a rotating disk on the fluid. The paper further assumes that the fluid layer that adheres to the aft of the fan due to the curved blade width absorbs some momentum resulting in reduction of the exerted momentum on the fluid by the fan. Axial velocity distribution for a ceiling fan obtained from the present analysis is found to be in good agreement with the available experimental as well as simulation data, which validates the two aforementioned assumptions.

Appendix: Derivation of $\nabla\times\left(\bar{r}\times\bar{V}\right)=-3\bar{V}$

Curl of $\bar{r}\times\bar{V}$ may be expressed by the vector identity [12]

$$\nabla\times\left(\bar{r}\times\bar{V}\right)=\bar{r}\left(\nabla\cdot\bar{V}\right)-\bar{V}\left(\nabla\cdot\bar{r}\right)+\left(\bar{V}\cdot\nabla\right)\bar{r}-\left(\bar{r}\cdot\nabla\right)\bar{V}\,. \tag{A.1}$$

By Taylor's theorem

$$d\bar{V}=dx\left(\partial\bar{V}/\partial x\right)+dy\left(\partial\bar{V}/\partial y\right)+dz\left(\partial\bar{V}/\partial z\right)\text{ + Higher order terms} \tag{A.2}$$

which in vector notation contracts to

$$d\bar{V}=d\bar{r}.\nabla\bar{V}\text{ + HOT.} \tag{A.3}$$

On integration Eq. (A.3) yields

$$\bar{V}=\bar{r}.\nabla\bar{V}-\int\bar{r}.d\left(\nabla\bar{V}\right)\text{ + HOT} \tag{A.4}$$

which can be re-written as

$$\bar{V}=\bar{r}.\nabla\bar{V}\text{ + HOT} \tag{A.5}$$

being the integral is also a higher order term. There $\nabla\cdot\bar{V}=0$ for incompressible flow, $\nabla\cdot\bar{r}=3$, $\left(\bar{V}.\nabla\right)\bar{r}=\bar{V}$ by expansion and contraction of its left side as Eqs. (A.2)-(A.3) and $\left(\bar{r}.\nabla\right)\bar{V}=\bar{V}$ from Eq. (A.5) by neglecting HOT. Hence Eq. (A.1) reduces to

$$\nabla\times\left(\bar{r}\times\bar{V}\right)=-3\bar{V} \tag{A.6}$$

References

1 Chiang, H.C., Pan, C.S., Wu, H.S. and Yang, B.C. Measurement of flow characteristics of a ceiling fan with varying rotational speed. Proceedings of Clima WellBeing Indoors, Helsinki, 2007, 1-8.

2 Zhao, R. and Li, J. The effective use of simulated natural air movement in warm environments. Indoor Air, 2004, 14, 46-50.

3 Sun, S.F., Ding, R.Y., Zhao, R.Y. and Xu, W.Q. Experimental study on unsteady air terminal. Proc. Int. Conference on Healthy Buildings, Singapore, 2003, 465-470.

4 Li, H., Chen, X., Ouyang, Q. and Zhu, Y. Wavelet analysis on fluctuating characteristics of airflow in building environments. Proceedings of Indoor Air, Beijing, 2005, 160-164.
5 Ouyang, Q., Li, H.J., Dai, W., Chen, X.C. and Zhu, Y.X. The differences and connections between the dynamic characteristics of natural and mechanical wind in built environment. Proceedings of International Conference on Indoor Air Quality and Climate, Beijing, 2005, 285-290.
6 Barna, P.S. Equilibrium of flow in axial flow fans designed for constant lift-drag ratio. Proc. First Australasian Conference on Hydraulics and Fluid Mechanics, Pergamon Press, 1964, 149-157.
7 Aynsley, R. Circulating fans for summer and winter comfort and indoor energy efficiency. Environment Design Guide, TEC, 2007, 25, 1-10.
8 Batchelor, G.K. *An Introduction to Fluid Dynamics*. Cambridge University Press, London, 1967.
9 Schlichting, H. and Gersten, K. *Boundary Layer Theory*. 8th Edition, Springer, Heidelberg, 2000.
10 Parker, D.S., Callahan, M.P., Sonne, J.K. and Su, G.H. Development of a high efficiency ceiling fan. The Gossamer Wind. Florida Solar Energy Center, FSEC-CR-1059-99, Cocoa, Florida, 1999.
11 Momoi, Y., Sagara, K., Yamanaka, T. and Kotani, H. Modeling of ceiling fan based on velocity measurement for CFD simulation of airflow in large room. 9th Int. Conference on Air Distribution in Rooms, Coimbra, 2004, 1-6.
12 Hildebrand, F.B. *Advanced Calculus for Applications*. Prentice-Hall, Inc. New Jersey, 1976.

Notation

δ thickness of fluid layer
ε dissipation rate of k
h fraction of blade width (= bc, Fig.2)
k turbulent kinetic energy
$\bar{\ell}$ line vector
$\bar{M}$ angular momentum vector
ω rotational speed of the fan
ϕ blade angle
r, θ, z radial, circumferential and axial directions
r_b radial spread of the flow
r_o radial distance of the blade tip
r_1 radius of fan bottom cover
R normalized r-coordinate (= r/r_b)
$\bar{S}$ area vector
τ_w shear stress on the blade surface
$\bar{V}$ velocity vector
V_c axial velocity on z-axis
V_m maximum axial velocity on radial plane
V_n normalized axial velocity [$=(V_z - V_c)/(V_m - V_c)$]
V_θ circumferential velocity
V_z axial velocity of the fan