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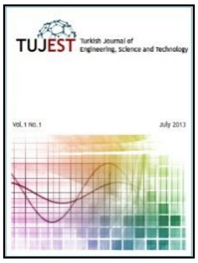

Unsteady Graphene Oxide Nanofluid Flow between Two Moving Plates: Analytical Solution

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Abstract

In this study we have used analytical approach to investigate unsteady nanofluid flow between two moving parallel plates. The effect of Graphene-Oxide (GO) nanoparticles solid volume fraction on temperature and velocity profiles have been discussed through some plots. The effect of moving parameter on velocity profiles have been shown for cases plates moving apart and moving together. Nusselt number have been achieved for various type of nanoparticles, solid volume fraction and Eckert number. It has been understood that by changing the solid volume fraction in constant Eckert number heat transfer enhancement will be achieved. The comparison of current solution with numerical one assures us about validity and accuracy of solution.

1. Introduction

Nanofluid ,a name conceived by Dr. Choi, in Argonne National Laboratory to describe a fluid in which nanometer-sized particle are suspended[1]. Nanoparticles have unique properties, such as large surface area to volume ratio, and lower kinematic energy which can be exploited in various applications [2, 3].

Graphene was found to display high quality electron transport at room temperature. Theoretical study was performed on determination of thermal conductivity of graphene and suggests that it has unusual thermal conductivity [4].

After that an experimental study was carried out on determination of thermal conductivity of graphene and 5300 W/mK was measured for thermal conductivity of single layer graphene. [5] Nanofluids are having wide area of application in electronic and cooling industry. Hydrogen exfoliated graphene (HEG) dispersed deionized (DI) water, and ethylene glycol (EG) based nanofluids were developed by Tessy Theres et, al [6]. Thermal conductivity and heat transfer properties of these nanofluids were systematically investigated. A 0.05% volume fraction of f-HEG dispersed DI water based nanofluid shows an enhancement in thermal conductivity of about 16% at 25°C and 75% at 50°C. The enhancement in Nusselts number for these nanofluids is more than that of thermal conductivity.

There are four possible mechanisms in nanofluids contribute to thermal conduction: a-ballistic nature of heat transport in nanoparticles, b- Brownian motion of nanoparticles, c- liquid layering at the liquid/particle interface, and d- nanoparticle clustering in nanofluids. The Brownian motion of nanoparticles is too slow to directly transfer heat through nanofluid; however, it could have an indirect role to produce a convection like micro environment around the nanoparticles and particle clustering to increase the heat transfer [7].

Squeezing flows have many applications in food industry, especially in chemical engineering. The determination of squeeze flow characteristics has attracted the attention of several investigators due to its importance in the practical problems of improving the performance of hydraulic machine elements, food industry, chemical engineering, polymer processing, compression, and injection molding [8-12].

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In this study, the Reconstruction of Variational Iteration Method [13], is applied to find the semi-analytical solutions of nonlinear differential equations governing the problem of unsteady nanofluid flow between two moving plates and heat transfer. The effects of the moving parameter, type of nanoparticle, the nanofluid volume fraction and Eckert number on Nusselt number are investigated.

2. Mathematical Formulation

The unsteady flow and heat transfer in a two-dimensional nanofluid between two infinite parallel plates is considered in this study. Figure.1 shows the problem schematic. The two plates are placed at $z = \pm l(1-\alpha t)^{1/2} = \pm h(t)$ for $\alpha > 0$, and moved until they touch $t = 1/\alpha$. For $\alpha < 0$ the two plates are separated.

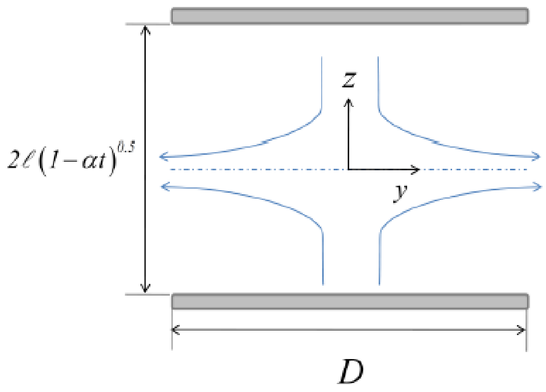

Figure.1. Geometry of problem

The viscous dissipation effect and heat generation due to friction caused by shear in the flow, is retained. This behavior occurs at high Eckert number (<<1). The Eckert number expresses the relationship between a flow's kinetic energy and enthalpy. The fluid is a water based nanofluid containing Graphene Oxide. The nanofluid is a two component mixture with the following assumptions: Incompressible; No-chemical reaction; Negligible viscous dissipation; Negligible radiative heat transfer; Nano-solid-particles and the base fluid are in thermal equilibrium and no slip occurs between them.The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \tag{4}$$

Where u and v are the velocities in the x and y directions, respectively. Effective density (ρ_{nf}), the effective dynamic viscosity (μ_{nf}), effective heat capacity $(\rho_{nf})_{nf}$ and the effective thermal conductivity k_{nf} of the nanofluid are defined as [14]:

$$\rho_{nf} = \rho_f(1-\phi) + \rho_s\phi_s$$
$$(\rho C_p)_{nf} = (\rho C_p)_f(1-\phi) + (\rho C_p)_s$$
$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{5}$$
$$\frac{k_{ns}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)}$$
$$v_{nf} = \frac{\mu_f}{\rho_{nf}}$$

The relevant boundary conditions for the problem are:

$$y = h(t) \rightarrow v = v_w = \frac{dh}{dt}, \quad T = T_H \tag{6}$$
$$y = 0 \rightarrow v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0$$

We can simplify above equations by introducing following parameters:

$$\eta = \frac{y}{\left[l(1-\alpha t)^{1/2}\right]}, \quad u = \frac{\alpha x}{[2(1-\alpha t)]} f'(\eta), \quad v = -\frac{\alpha l}{\left[2(1-\alpha t)^{1/2}\right]} f(\eta), \tag{7}$$
$$\theta = \frac{T}{T_H}, \quad A = (1-\varphi) + \varphi\frac{\rho_s}{\rho_f}, \quad B = (1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}, \quad C = \frac{k_{nf}}{k_f}$$

The above parameters are substituted into equations. (2) and (3). Then the pressure gradient is eliminated from the resulting equations:

$$f^{(IV)} - SA(1-\varphi)^{2.5}(\eta f''' + 3f'' + f'f'' - ff''') = 0 \tag{8}$$

Using equation (7), Equations (3) and (4) simplify to following equations:

$$\theta'' + \Pr S\left(\frac{B}{C}\right)(f\theta' - \eta\theta') + \frac{\Pr Ec}{C(1-\varphi)^{2.5}}\left(f''^2 + 4\delta^2 f'^2\right) = 0 \tag{9}$$

With the following boundary conditions:

$$f(0) = 0, \quad f''(0) = 0, \quad f(1) = 1, \quad f'(1) = 0, \quad \theta'(0) = 0, \quad \theta(1) = 1 \tag{10}$$

Where S is moving parameter, $\Pr$ is the Prandtl number and Ec is the Eckert number, Which are defined as:

$$S = \frac{\alpha l^2}{2v_f}, \quad \Pr = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f}, \quad Ec = \frac{\rho_f}{(\rho C_p)_f}\left(\frac{\alpha x}{2(1-\alpha t)}\right)^2, \quad \delta = \frac{l}{x} \tag{10}$$

moving parameter S describes the movement of the plates. $S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together (also called squeezing flow).

3. Solution Procedure

In the following section, an alternative method for finding the optimal value of the Lagrange multiplier by the use of the Laplace transform will be investigated. suppose x, t are two independent variables, consider t as the principal variable and x as the secondary variable. if $u(x,t)$ is function of two variables x and t, when the Laplace transform is applied with t as a variable, definition of Laplace transform is:

$$L(u(x,t);s)=\int_0^\infty e^{-st}u(x,t)dt \tag{11}$$

We have some preliminary notations as:

$$L\left(\frac{\partial u}{\partial t};s\right)=\int_0^\infty e^{-st}\frac{\partial u}{\partial t}dt=sU(x,s)-u(x,0) \tag{12}$$

$$L\left(\frac{\partial^2 u}{\partial t^2};s\right)=s^2U(x,s)-sU(x,s)-u_t(x,0) \tag{13}$$

$$U(x,s)=L(u(x,t);s) \tag{14}$$

We often come across functions which are not the transform of some known function but then they can possibly be as a product of two function. Thus we may be able to write the given function as $U(x,s)$, $V(x,s)$ where $U(s)$ and $V(s)$ are known to the transform of the function $u(x,t)$, $v(x,t)$, when the Laplace transform is applied to t as a variable, respectively; then $U(x,s)$, $V(x,s)$ is the Laplace Transform of $\int_0^t u(x,t-\varepsilon)v(x,\varepsilon)d\varepsilon$:

$$L^{-1}(U(x,s),V(x,s))=\int_0^t u(x,t-\varepsilon)v(x,\varepsilon)d\varepsilon \tag{15}$$

To facilitate our discussion of Reconstruction of Variational Iteration Method (RVIM), introducing the new linear or nonlinear function $h(u(x,t))=f(x,t)-N(u(x,t))$ and considering the new equation, rewrite $h(u(x,t))=f(x,t)-N(u(x,t))$ as:

$$L(u(t,x))=h(t,x,u) \tag{16}$$

Now, for implementation the correctional function of VIM based on new idea of Laplace transform, applying Laplace Transform to both sides of the above equation so that we introduce artificial initial conditions to zero for main problem, then left hand side of equation after transformation is featured as:

$$L[L\{u(x,t)\}]=U(x,s)P(s) \tag{16}$$

Where $P(s)$ is polynomial with the degree of the highest order derivative of linear operator :

$$L[L\{u(x,t)\}]=U(x,s)P(s)=L[h\{(x,t,u)\}] \tag{17}$$

$$U(x,s)=\frac{L[h\{(x,t,u)\}]}{P(s)} \tag{18}$$

Suppose that $D(s)=1/P(s)$, Using the convolution theorem, Taking the inverse Laplace transform on both side of Equation.(22),

$$u(x,t)=\int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \tag{19}$$

$$u_0(x,t)+\int_0^t d(t-\varepsilon)h(x,\varepsilon,u)d\varepsilon \tag{20}$$

And $u_0(x,t)$ is initial solution with or without unknown parameters. In absence of unknown parameters, $u_0(x,t)$ should satisfy initial boundary conditions.

4. Results and Discussions

In this section we will discuss about the obtained results of squeezing Go-Water nanofluid flow between infinite parallel plates problem for various solid volume fraction and moving parameter. The physical properties of Gaphene Oxide- Water nanofluid can be found in table.1.

Table1. Thermo physical properties of water and GO nanoparticle[].

	$\rho(kg/m^3)$	$C_p(j/kgk)$	$k(W/m.k)$
Pure water	997.1	4179	0.613
Graphene Oxide	1800	717	5000

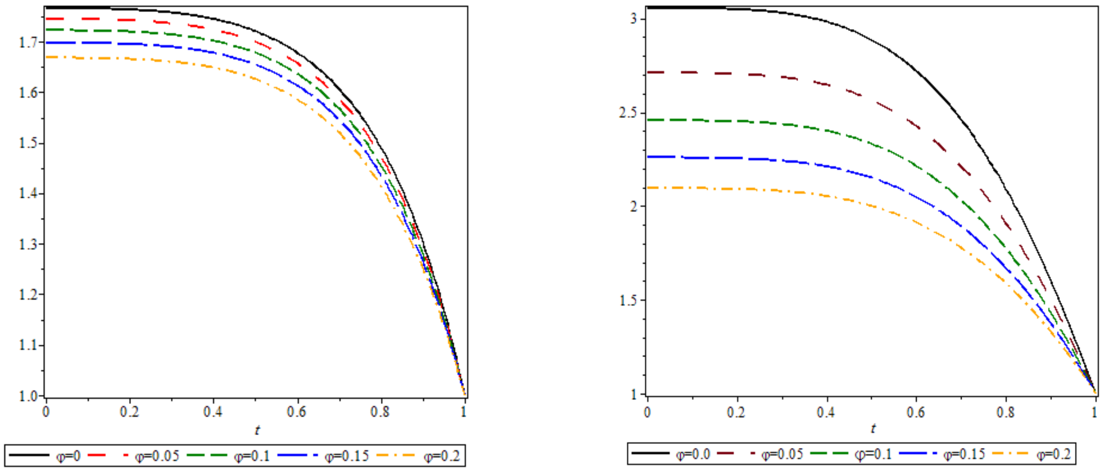

Figure 2. Temperature profile for various solid volume fraction when $\delta = 0.1, \quad \Pr = 6.2, \quad Ec = 0.2$ a- $S = 1$, (plates moving apart) b- $S = -1$, (squeezing flow).

Figure.2a,b show the effect of solid volume fraction on temperature profiles (θ) when $\delta = 0.1, \quad Ec = 0.2, \quad \Pr = 6.2$ for two different case: a- when plates moving apart ($S > 0, S = 1$) and b- when plates moving together ($S < 0, S = -1$). Solid volume fraction value changes from 0.0 (pure water) to 0.2. As it can be illustrated in Figure.2 the variation of Graphene-Oxide solid volume fraction is more effective on temperature profile for second case (squeezing flow). Figue.3 show velocity profile for various moving parameter when $\varphi = 0.1, \quad \delta = 0.1$. It is important to note that in point $\eta = 0.5$ velocity has constant value and moving parameter has no effect on it.

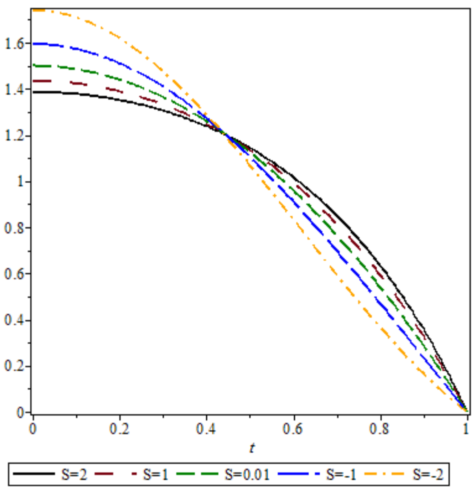

Figure.3. Velocity profile for various moving number when $\varphi = 0.1, \quad \delta = 0.1$

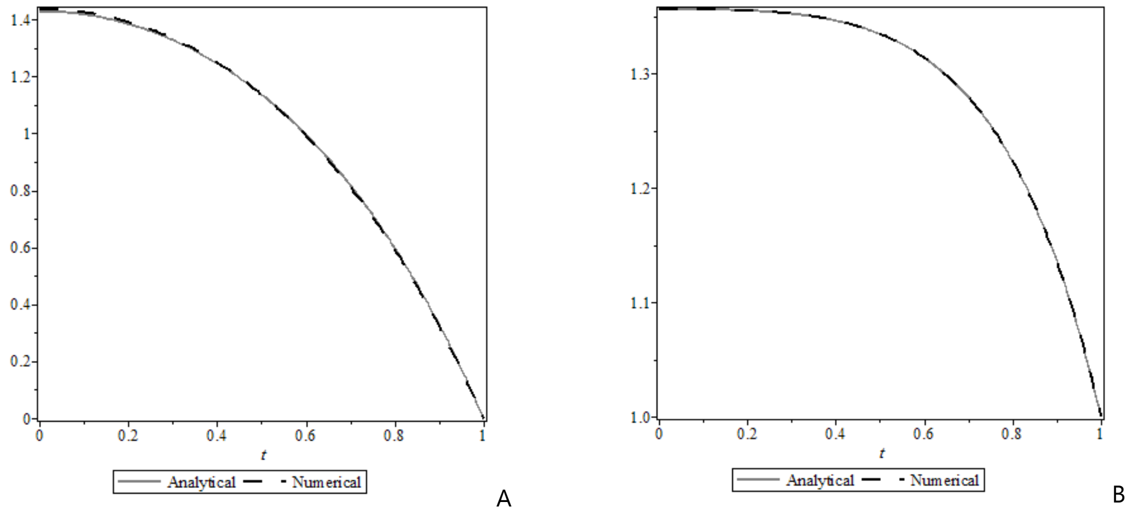

Figure.4. Comparison of analytical results and numerical ones when $S = 1, \quad \varphi = 0.05, \quad \delta = 0.1, \quad \Pr = 6.2, \quad Ec = 0.1$ for a- *Velocity* b- *Temperature*

In Figure.4. the comparison of approximate results with numerical solutions have been presented when $S = 1, \quad \varphi = 0.05, \quad \delta = 0.1, \quad \Pr = 6.2, \quad Ec = 0.1$. In first plot, the velocity profile has been shown and in second plot we can see the results comparison for temperature profile. As it can be seen numerical results have good agreement with obtained results for both temperature and velocity.

Table.2. Nusselt number for various nanoparticle, solid volume fraction and Ec when $S = 1, \quad \delta = 0.1, \quad \Pr = 6.2$

	φ	Eckert	Nusselt
Graphene Oxide	$\varphi = 0.1$	$Ec = 0.1$	2.57202
		$Ec = 0.2$	5.14405
	$\varphi = 0.2$	$Ec = 0.1$	3.54872
		$Ec = 0.2$	7.09734
Aluminium Oxide	$\varphi = 0.1$	$Ec = 0.1$	2.58103
		$Ec = 0.2$	5.1658
	$\varphi = 0.2$	$Ec = 0.1$	3.56154
		$Ec = 0.2$	7.12308
Titanium Oxide	$\varphi = 0.1$	$Ec = 0.1$	2.57391
		$Ec = 0.2$	5.14794
	$\varphi = 0.2$	$Ec = 0.1$	3.54059
		$Ec = 0.2$	7.08118
Silver	$\varphi = 0.1$	$Ec = 0.1$	2.66528
		$Ec = 0.2$	5.33057
	$\varphi = 0.2$	$Ec = 0.1$	3.70816
		$Ec = 0.2$	7.41632

Table.2 presents the effect of Eckert number, nanoparticles type, and its solid volume fraction on Nusselt number. Four different type of nanofluid have been presented (Graphene Oxide, Aliminium Oxide, Titanium Oxide, Silver). More heat transfer enhancement is obtained by increasing the solid volume fraction and Eckert for all cases but for same value of

φ, Ec the maximum amount of Nusselt number will be obtained by choosing Silver (Ag) as the nanoparticles. As it can be seen Eckert number has strong effect on Nusselt number. Another consequence that can be achieved from table is by increasing the solid volume fraction from 0.1 to 0.2 we can increase the Nusselt number about 20 or 30% for each case.

5. Conclusion

In this paper, the heat transfer in the unsteady nanofluid flow between two moving parallel plates was investigated using RVIM. The effect of solid volume fraction on heat transfer enhancement for case plates moving apart have been studied as well as squeezing flow. Velocity profiles for various moving number have been also obtained. The effect of moving parameter on Nusselt number has been investigated for different nanoparticles (Graphene Oxide, Aliminium Oxide, Titanium Oxide, Silver) with different solid volume fraction and Eckert number. The comparison of obtained results with numerical solutions assures us about the validity and accuracy of the current study.

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