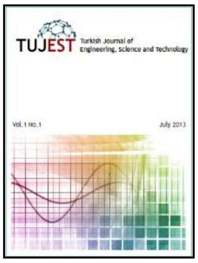
Turkish Journal of Engineering, Science and Technology 01 (2013) p11-18

Turkish Journal of Engineering, Science and Technology

journal homepage: www.tujest.com



Assessment On The Performance Of Pelton Turbine Test Rig Using Response Surface Methodology

Alok Ku Nanda[a], Soumya Dash[b], R. Bhima Rao[c], Satya Sai Srikant[d,]*

[a, b, c] Aryan Institute of Engineering and Technology, (Biju Pattnaik University of Technology), Bhubaneswar, Odisha, India
[d] SRM University, Modinagar, Delhi, India

Article Info

Article history:
Received 4 July 2013
Accepted 8 July 2013
Available online 23 July 2013

Keywords:
Pelton Turbine
Impulse Turbine
RSM
Performance
Discharge
Power

Abstract

This paper deals to assess the performance of the Pelton wheel turbine with different range of rotational speeds and varying loads using surface response methodology software Design Expert DX6 and their results achieved from the present investigation are discussed. The test results reveals that the predicted value and the observed value for turbine shaft rotation in rpm are indicating a very good fit for the response function equation. Similarly the predicted values and the observed values for discharge, input and output power are found same. It is also found that the predicted value [29.46%] and the experimental value [29.52%] are very good fit for the efficiency of Pelton turbine.

1. Introduction

Turbine is the hydraulic machine which converts first hydraulic energy into mechanical energy and then finally converts it into electric energy. Turbines are two types, reaction and the impulse, the difference being the manner of head conversion. In the reaction turbine, the fluid fills the blade passages, and the head change or pressure drop occurs within the runner. An impulse turbine first converts the water head through a nozzle into a high velocity jet, which then strikes the buckets at one position as they pass by. The runner passages are not fully filled, and the jet flow past the buckets is essentially at constant pressure. Impulse turbines are ideally suited for high head and relatively low power. The Pelton turbine used in this experiment is an impulse turbine. In 1870 L A Pelton invented the impulse turbine and hence the impulse turbines are known as Pelton turbine. This Pelton is an impulse turbine which flows tangentially. The water tangentially strikes the double hemispherical cup (shape of bucket) on the runner. The energy available at the inlet of the turbine is only kinetic energy [1-2]. The pressure at the inlet and outlet of the turbine is atmosphere. The main aim of this paper is to assess the performance of the Pelton wheel turbine with different range of rotational speeds and varying loads using surface response methodology.

Response Surface Methodology (RSM) is a collection of statistical and mathematical methods that are useful for modeling and analyzing problems. Response Surface Methodology (RSM) is a useful tool which allows one to obtain appropriate data that can be analyzed to arrive at objective conclusions and determine the optimum conditions through a relatively smaller number of systematic experiments [3, 4]. The RSM also gives the relationship between the controllable input parameters and the response surfaces [3-7]. Thus the response surface methods (RSM) are being used by several authors for finding the optimized conditions with controlled input variables to achieve the output in short time. It is known that almost all the researchers and academicians are widely accepted to use the RSM in many fields such as

*Corresponding Author:
S.S. Srikant, E-mail: satya.srikant@gmail.com

mineral and metallurgical engineering, material science, food research etc [3-7]. Similarly, there are several publications on optimization of process parameters as well as study on the dynamics of water flow etc in the area of Pelton turbine. However, publications are limited in the area of optimization of process parameters of Pelton turbine using Response Surface Methodology. Thus it is evident from the literature that optimizing the process variable using RSM for formation of Pelton turbine is so far not attempted. In view of this, the present investigation deals with the use of RSM software Design Expert DX6 to optimize the performance of Pelton wheel turbine with different rotational speed including determining the output power of Pelton Wheel Turbine, to find out the efficiency of the Turbine as well as the operation of a Pelton Wheel Turbine and their results achieved from the present investigation are discussed.

2. Materials And Methods

The Pelton turbine used in the present investigation is a laboratory test rig designed and fabricated and supplied by M/s Tech Ed, Bangalore for engineering graduate and post graduate students laboratory experimental purpose. However some of the aspects of the turbine used in the present investigation are described under the following sub headings.

2.1. Experimental

The Pelton turbine used in the present investigation is similar to the conventional turbine which consists of three basic components, a stationary inlet nozzle, a runner and a casing. The runner consists of multiple buckets mounted on a rotating wheel. The inlet nozzle jet strikes the buckets and imparts momentum. The buckets are shaped in a manner to divide the flow in half and turn its relative velocity vector nearly 180°. The turbine experimental facility supplied was fitted with centrifugal pump set, weight with variable loads, spring balance, rope used weight / load, tachometer, pressure meters, sump tank etc., arranged in such a way that the whole unit works as re-circulating water system. The centrifugal pump set supplies the water from sump tank to the turbine through control valve. The loading of the turbine is achieved by rope brake drum connected to spring balance. The design parameters of the turbine are given in Table 1.

Table1 Design Parameters of Pelton turbine test rig

Factors	Actual Value
Motor Specification	50Hz line , AC, 5 hp , 440 Volts
Spring Balance	5 Kg
Number of buckets (vanes)	16
Cross sectional area of sump tank	2.88 m^2
Fluid Density (H_2O)	1000 Kg/m^3
Venturimeter Size at inlet	50 mm
Throat diameter	30 mm
Break drum diameter (D)	210 mm
Rope diameter	20 mm
Weight of rope and hanger	1Kg
Value of "K"	5.02×10^{-3}
Runner diameter	260 mm
Nozzle diameter	25 mm

2.2. Operating procedure

The volume of the present turbine sump tank is 2.88 $m^{3.}$ However, the water used in the sump tank was up to $3/4^{th}$ of its volume. Initially the pump was started to run the runner. The speed of the runner was controlled by varying weights of different loads. The unit was designed with 16 numbers of buckets (vanes). The water strikes the vanes (buckets) through the nozzle and spear with a theoretical velocity of 23.7585 m/sec in pipe. The hydraulic energy is converted into mechanical energy which is transmitted through the shaft by the rotation of the runner. The rotational speed of the shaft was measured with the help of a tachometer. Experiments were carried out with different range of rotational speeds and varying loads and nozzle jet pressure keeping constant.The spring balance reading, pressure gauge reading and the venturimeter difference readings were recorded to determine the discharge, input and output power.

Equations used for Pelton turbine

Total Head	$H = 10 \times P$ (m of water)
Velocity of water in pipe (m /sec)	$V = (2\,g\,H)^{0.5}$
Differential pressure of manometer	$h = (P_1-P_2)\times10$ (m of water)
Discharge (m^3/s)	$Q = K\sqrt{h}$
Power input (kW)	$P_{in}= \gamma\,Q\,H$
Power output (kW)	$P_{out} = (2\pi N W_3 R_e \times 9.81) / 60{,}000$
Effective radius	$R_e = 0.5\,D_b + D_r$
Efficiency	η % = (power output/power input) × 100

Where ρ_w = Density of water and h = Differential pressure of manometer (m); P = Pressure gauge reading (kg/cm^2); P_1 = Pressure reading at venturimeter inlet; P_2 = Pressure reading at venturimeter throat; W_1 = weight on hanger (kg); W_2 = Adjustable spring balance reading (kg); V= Velocity of water in pipe (m/sec); N = speed of turbine shaft in rpm; D_b = Diameter of brake drum; D_r = Diameter of rope; γ = specific gravity of water i.e. 9.81 KN/m^2

2.3. Design of experimental procedure

1. Designing of experiments for adequate and reliable measurement of the response.
2. Developing a mathematical model of the second-order response surface with the best fittings.
3. Finding the optimal set of parameters that produce a maximum or minimum value of response.
4. Representing the direct and interactive effects of process parameters through two or three dimensional plots.

If all controlled variables are assumed to be measurable then the response surface can be expressed as:

$$\boldsymbol{y = f\ (x_1, x_2, x_3 \cdots\cdots\cdots\ x_k)} \quad \textbf{(1)}$$

where y is the output and x_i the variables of action called factor.

The goal of this aspect of the study was to optimize the response variable y. The independent variables are assumed to be continuous and controllable by the experiments with negligible errors. A reasonable approximation for the true functional relationship between independent variables and the response surface is desired. A second-order model was usually used in response surface methodology [4, 8, 9]

$$\boldsymbol{y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=2}^{k} \beta_{ij} x_i x_j + \varepsilon} \quad \textbf{(2)}$$

where x_1, x_2,..., x_k were input factors which influence the response y; β_0, β_{ii} (i=1, 2, ..., k), β_{ij} (i=1, 2, ..., k; j=1, 2, ..., k) were unknown parameters and ε was a random error. The β coefficients, which should be determined in the second-order model, are obtained by the least square method. Generally equation 2 can be written in matrix form [4, 8, 9]:

$$\boldsymbol{Y = bX + \in} \quad \textbf{(3)}$$

where Y is defined to be a matrix of measured values, X to be a matrix of independent variables. The matrices b and ∈ consist of coefficients and errors, respectively. The solution of equation (3) can be obtained by the matrix approach [4, 9]

$$\boldsymbol{b = (X'X)^{-1}X'Y} \quad \textbf{(4)}$$

where X′ is the transpose of the matrix X and $(X'.X)^{-1}$ is the inverse of the matrix X′.X.

Considering the effects of the main factors and also the interactions between two-factor sets, equation 2 takes the form:

$$\boldsymbol{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 + \beta_{23} x_2 x_3} \quad \textbf{(5)}$$

where y is the predicted response, β_0 is a model constant; x_1, x_2 and x_3 are independent variables; β_1, β_2 and β_3 are linear coefficients; β_{12}, β_{23} and β_{13} are cross product coefficients; and β_{11}, β_{22} and β_{33} are the quadratic coefficients [4,9,10]. The various coefficients, β_1, β_2 , β_3 , β_{11}, β_{22} , β_{33}, β_{12}, β_{23} , β_{13} can be found out from the experimental results by using Design Expert DX6 software v.7 called ANOVA on applying the Quadratic (square) method.

3. Results And Discussion

The performances of the Pelton wheel turbine with different ranges of rotational speeds are given in Table 2 and Fig 1 and Fig 2. As expected, it is seen from the Table 3 that the rotational speeds (rpm) is gradually decreasing with increase in weight W (Kg) and tension. This can clearly be seen from Fig 1 that with increasing the net load, the speed (rpm) is decreasing.

Thus it can be concluded that net load is inversely proportional to the speed (rpm). It is observed that with the effect on increasing the net load for discharge and Power input has no significant effect where as the Power output marginally increases from 0.15 to 1.01 kW with increasing the net load from 1 to 8.63 Kg. The observation is similar to the efficiency of Pelton turbine. The efficiency on the performance of the Pelton turbine test rig is found 4.2% at 1 Kg of net load which has been increased to 29.5% at 8.63 Kg of net load. This observation can clearly be seen from Fig 2 that with increasing net load of the performance of the Pelton turbine efficiency is increasing. This is due to a fact that the turbine is as the hydraulic machines which converts hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted in to electric energy. The Pelton wheel is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmosphere. The turbine is used for high head.

Table 2. Experimental data obtained from Pelton turbine

Net Load W_3	Speed N, rpm	Pressure gauge P	P_1	P_2	Discharge, Q	P_{in}	P_{out}	Efficiency, %
1.00	1204	3	3.32	2.71	0.012398	3.648874	0.154671	4.238869
1.86	1154	2.98	3.30	2.69	0.012398	3.624548	0.275741	7.607593
2.70	1116	2.96	3.28	2.68	0.012296	3.570591	0.387089	10.84102
3.49	1086	2.94	3.27	2.67	0.012296	3.546465	0.486898	13.7291
4.25	1044	2.92	3.26	2.65	0.012398	3.551571	0.569996	16.04912
5.00	1018	2.90	3.24	2.64	0.012296	3.498214	0.653883	18.69192
5.79	987	2.89	3.22	2.62	0.012296	3.486151	0.734139	21.05872
6.75	966	2.89	3.22	2.62	0.012296	3.486151	0.837391	24.02051
7.70	932	2.88	3.21	2.61	0.012296	3.474088	0.921713	26.53108
8.63	919	2.86	3.20	2.60	0.012296	3.449963	1.018514	29.52246

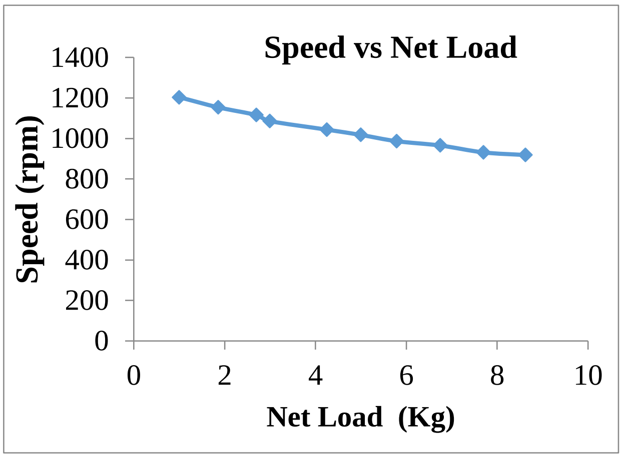


Fig 1. Effect of net load on speed, rpm

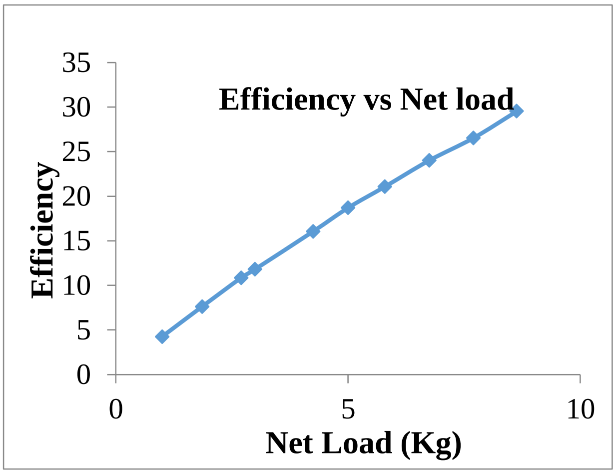
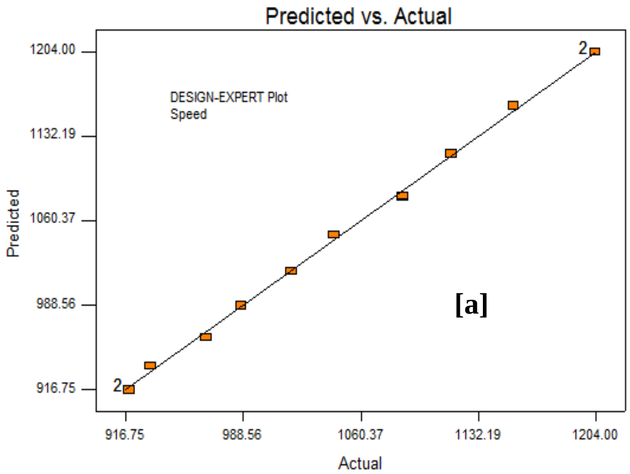
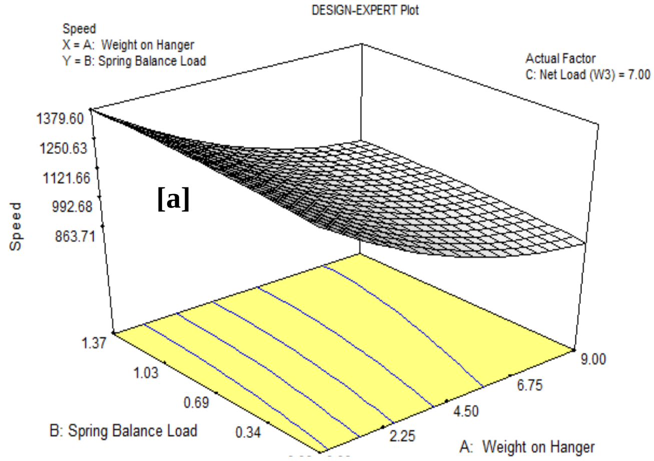


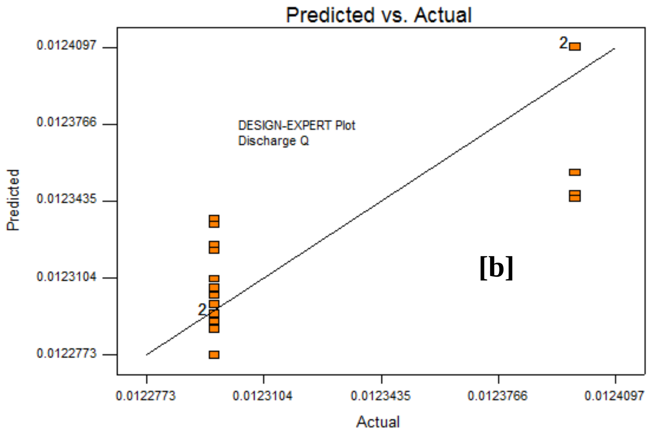
Fig 2. Effect of net load on efficiency, %

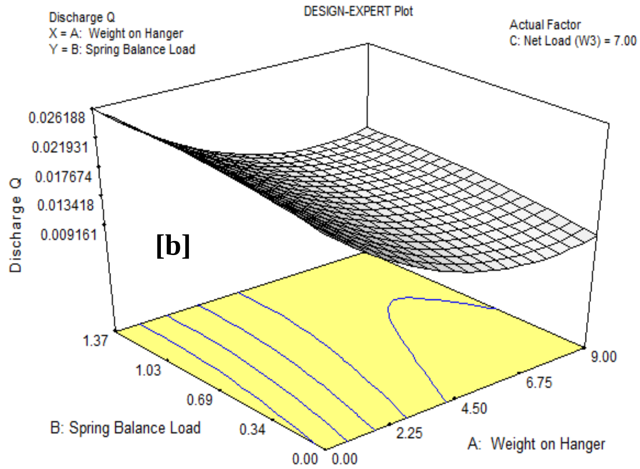
3.1. *Response Surface Method*

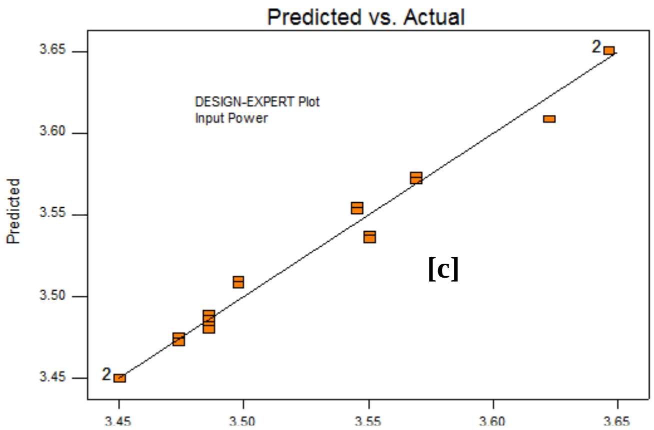
The response surface method (RSM) used for Pelton experiment for 19 experiments with DX6- RSM software for the efficiency and power generation with fixed pressure guage and the input variables weight on hanger (W1), spring balance load (W2) and net load (W3) with various combinations are discussed in the following with respect to Fig.3[left column] and Fig.4[right column].

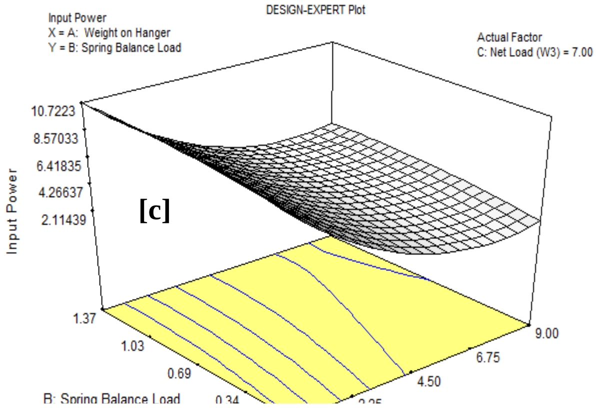

Predicted vs. Actual
DESIGN-EXPERT Plot
Speed
Predicted
Actual
1204.00
1132.19
1060.37
988.56
916.75
[a]

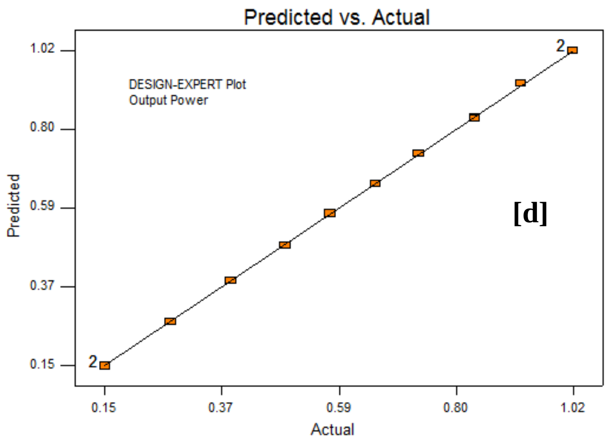

DESIGN-EXPERT Plot
Speed
X = A: Weight on Hanger
Y = B: Spring Balance Load
Actual Factor
C: Net Load (W3) = 7.00
1379.60
1250.63
1121.66
992.68
863.71
Speed
[a]
B: Spring Balance Load
A: Weight on Hanger

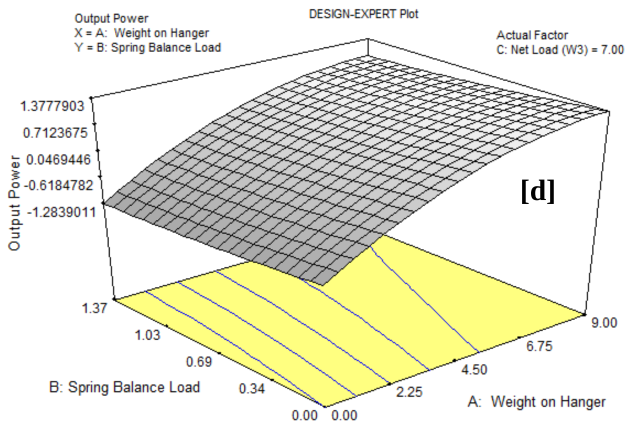

Predicted vs. Actual
DESIGN-EXPERT Plot
Discharge Q
Predicted
Actual
0.0124097
0.0123766
0.0123435
0.0123104
0.0122773
[b]

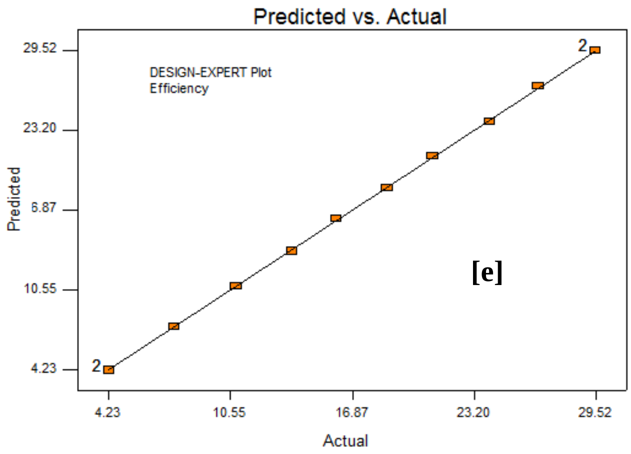

DESIGN-EXPERT Plot
Discharge Q
X = A: Weight on Hanger
Y = B: Spring Balance Load
Actual Factor
C: Net Load (W3) = 7.00
0.026188
0.021931
0.017674
0.013418
0.009161
Discharge Q
[b]
B: Spring Balance Load
A: Weight on Hanger


Predicted vs. Actual
DESIGN-EXPERT Plot
Input Power
Predicted
3.65
3.60
3.55
3.50
3.45
[c]


DESIGN-EXPERT Plot
Input Power
X = A: Weight on Hanger
Y = B: Spring Balance Load
Actual Factor
C: Net Load (W3) = 7.00
10.7223
8.57033
6.41835
4.26637
2.11439
Input Power
[c]


Predicted vs. Actual
DESIGN-EXPERT Plot
Output Power
Predicted
Actual
1.02
0.80
0.59
0.37
0.15
[d]


DESIGN-EXPERT Plot
Output Power
X = A: Weight on Hanger
Y = B: Spring Balance Load
Actual Factor
C: Net Load (W3) = 7.00
1.3777903
0.7123675
0.0469446
-0.6184782
-1.2839011
Output Power
[d]
B: Spring Balance Load
A: Weight on Hanger



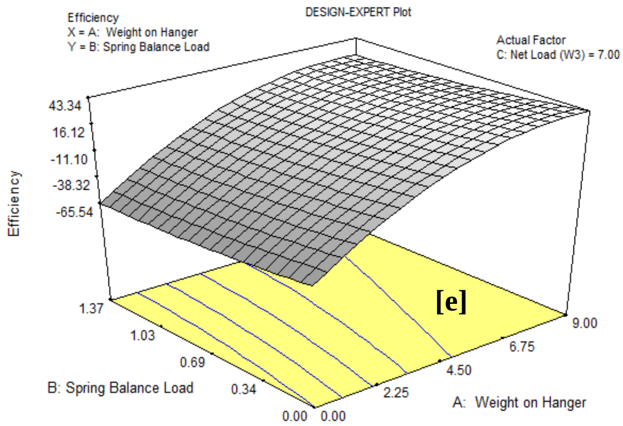


Fig 3 [Left Column] Optimized response surface plots for the experiments performed for Pelton turbine experiment showing (a) speed (rpm) (b) discharge Q (m3/ sec) (c) amount of input power (kW) (d) amount of output power (kW) (e) Efficiency (%)
Fig 4 [Right Column] Optimized response surface plots with 3-D view for Pelton turbine experiment showing (a) speed (rpm) (b) discharge Q (m3/ sec) (c) amount of input power (kW) (d) amount of output power (kW) (e) Efficiency (%)

3.1.1. Effect of variables on speed

It is explained earlier in the text that the net load is inversely proportional to the speed (rpm). The wheel is connected to a string which will be loaded with weights in order to apply a torque to the wheel. When the flow is set to a certain flow rate, the flow will rotate the wheel by means of transferring the momentum through the cups connected to the wheel. When the loads are increased, the wheel will be harder to rotate due to the increase in torque applied to the wheel and hence the speed reduces. The effect of variables on speed has been studied using response surface methodology.

The relationship between response and variables obtained of coded factors for speed in rpm is shown in equation 6.

$$y_1 = +570.06 - 192.53 * x_1 - 1095.11 * x_2 + 225.08 * x_1^2 - 764.82 * x_2^2 - 156.48 * x_3^2 \quad \textbf{(6)}$$

The experimental results and the predicted values obtained using these model response equations [4,5,6] are shown in Fig 3a. The predicted values and the observed values are indicating a very good fit (R^2 value of 0.9889) for speed in rpm for the response function equation. The optimized response surface plot with 3-D view showing for speed in rpm can clearly be seen in Fig. 4a. It is seen from the figure that with increasing the weight of hanger (W_1) and spring balance load (W_2), the speed is found to be decreasing.

3.1.2. *Effect of variables on Discharge Q (m^3/ sec)*

The relationship between response and second variables obtained of coded factors for discharge (m^3 /sec) occurred is shown in equation 7.

$$y_2 = - 0.015 - 0.0009692 * x_1 - 0.073 * x_2 + 0.013 * x_1^2 - 0.049 * x_2^2 - 0.014 * x_3^2$$

3.1.3. Effect of variables to generate input power (kW)

The relationship between response and variables obtained to generate the input power is shown in equation 8.

$$y_3 = -8.21 - 0.58 * x_1 - 31.43 * x_2 + 6.65 * x_1^2 - 21.10 * x_2^2 - 7.75 * x_3^2 \quad \textbf{(8)}$$

The predicted values and the experimental values shown in Fig 3c, are indicating a very good fit (R^2 value of 0.9914) to generate the input power for the given weight of hanger (W_1), spring balance load (W_2), its corresponding discharge and pressure gauge. The optimized response surface plot with 3-D view showing for mullite formation can clearly be seen in Fig. 4c. It is seen from the figure that input power varies in accordance to the discharge obtained as shown in Fig 3c

The experimental results and the predicted values obtained using these model response equations are shown in Figure 3b. The predicted values and the experimented values are indicating a very good fit (R^2 value of 0.8277) for discharge (m^3/ sec) in the response function equation. The optimized response surface plot with 3-D view showing for discharge (m^3/ sec) can clearly be seen in Fig. 4b. The discharge is the change of pressure obtained from venturimeter when the weight on hanger and spring balance load are varying [1-2].

3.1.4. Effect of variables to generate output power (kW)

The relationship between response and variables obtained to generate the input power is shown in equation 9.

$$y_4 = +2.17 + 0.82^* x_1 + 4.38^* x_2 - 1.28^* x_1^2 + 3.46^* x_2^2 + 1.37^* x_3^2 \quad \textbf{(9)}$$

The predicted values and the experimental values shown in Fig 3d, are indicating a very good fit (R^2 value of 0.9999) for the output power generation for the given net load (W_3) obtained by weight of hanger (W_1), spring balance load (W_2) along with its corresponding speed (rpm) and pressure gauge. The optimized response surface plot with 3-D view showing for output power can clearly be seen in Fig. 4d. It is observed output power varies in accordance to the speed and the net load (W_3) due to difference between weight of hanger (W_1) and spring balance load (W_2)

3.1.5. Effect of variables for efficiency

The relationship between response and variables obtained for efficiency is shown in equation 10.

$$y_5 = +110.05 + 25.84^* x_1 + 252.09^* x_2 - 63.05^* x_1^2 +182.74^* +71.34^* x_3^2 \quad \textbf{10)}$$

The predicted values and the experimental values shown in Fig 3e are indicating a very good fit (R^2 value of 0.9999) for the efficiency of Pelton turbine. This shows that the efficiency varies in accordance with the output power generation for the given net load (W_3) obtained by weight of hanger (W_1), spring balance load (W_2) along with its corresponding speed (rpm) and pressure gauge. The optimized response surface plot with 3-D view showing for output power can clearly be seen in Fig. 4e. It is found from the efficiency vs. speed graphs, that the predicted efficiency [29.46, %] is lower than the theoretical which is due to the fluid friction which reduces the kinetic energy of the flow. As the speeds increase, the efficiency increase too until to a certain point, it will decrease meaning that the efficiency of the wheel will decrease eventually and it is impossible to reach an increasing efficiency based on the theoretical value. However, in the present investigations, the unit could not run beyond 30% efficiency, which was specified in the technical manual of the test rig.

Results of actual and predicted values obtained on the performance of Pelton turbine are given in Table 3. This indicate that the data obtained from the experimental data and the data obtained from the response surface methodology are very much close with each other.

Table 3. Results of optimum values achieved on the performance of Pelton turbine

Factors	**Actual Value**	**Predicted Value**
Speed (N rpm)	1204	1203.12
Discharge (Q)	0.0123	0.01229
Input Power (kW)	3.65	3.65
Output Power (kW)	1.02	1.02
Efficiency (η %)	29.52	29.46

The predicted value [1203.12, rpm] and the observed value [1204, rpm] are indicating a very good fit for speed in rpm for the response function equation. Similarly the predicted values and the observed values for discharge, input and output power are found same. It is also found that the predicted value [29.46%] and the experimental value [29.52%] are very good fit for the efficiency of Pelton turbine.

4. Conclusions

The conclusions are drawn from the study on the assessment for the performance of Pelton turbine test rig using response surface methodology. The results of this study reveal that **t**he predicted value for rpm is 203.12 where as the observed value are 1204. It is also found that the predicted value [29.46%] and the experimental value [29.52%] are very good fit for the efficiency of Pelton turbine. Thus, the predicted values and the observed values for speed, discharge, input and output power are indicating a very good fit for the response function equation.

5. Acknowledgements

Er. Satya Sai Srikant and other two authors of the institute are thankful to the Dr (Prof.) R. Bhima Rao, Principal, Aryan Institute of Engineering and Technology Bhubaneswar for giving permission to utilize the infrastructural facilities to do carry out this research work.

References

[1] R K Bansal, (2010), A Textbook of fluid mechanics and hydraulic machines, Laxmi Publications, 9th Edition (2010), pp 857 - 861.

[2] Robertson, J.A. and Crowe, C.T. (1993), Engineering Fluid Mechanics, 5th Edition, Houghton Mifflin, Boston.

[3] M. Kincl, S. Turk, F. Vrecer, Application of experimental design methodology in development and optimization of drug release method, International Jr of Pharmaceutics. 291 (2005) 39–49.

[4] N. Aslan, Application of response surface methodology and central composite rotatable design for modeling and optimization of a multi-gravity separator for chromite concentration, Powder Technology. 185 (2008) 80–86.

[5] S.S Srikant, P.S Mukherjee, R Bhima Rao, (2012), Applications of response surface methodology on mullite formation from red sediment placer sillimanite using microwave energy**,** International Journal- Multi Disciplinary Edu Global Quest Quaterly); Vol 1(4), pp 104-117

[6] S. Ozgen, A. Yıldız, A. Çalışkan, E. Sabah, (2009), Modeling and optimization of hydrocyclone processing of low grade bentonites, Applied Clay Science 46 (3) Nov, pp 305–313

[7] N. Aslan and Y. Cebeci, (2007), Application of Box–Behnken design and response surface methodology for modelling of some Turkish coals, Fuel 86 , pp 769– 776.